OCR A Physics A-level

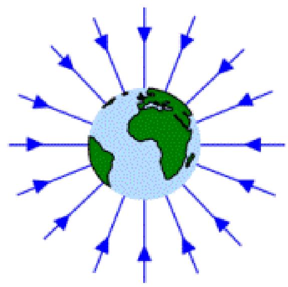
Topic 5.5: Gravitational fields

Notes

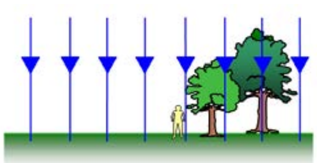
Gravitational fields

Gravitational field patterns

Gravity is a **universal attractive force**. It is considered to be a weak force, and has an **infinite range**. All objects with mass experience a gravitational field, which extends towards infinity. As the distance from the centre of mass of the object increases, the gravitational field strength decreases until it becomes infinite. Objects placed within the field will be **attracted towards** the **centre of mass** of the object. All objects with mass can be modelled as a point mass, where the point is the centre of mass of the object.



For a radial field, such as for a planet, when the gravitational field pattern is drawn, it has straight lines converging to the centre of mass. The lines are arrows pointing to the centre of mass, to show that gravity is an attractive force. The **closer** the lines are to each other, the **stronger** the field is at that point. The lines never cross each other.



Sometimes, a gravitational field can be modelled as **uniform**, such as looking at the surface of a planet on a small scale. Here, the field strength is equal at all positions, so the field is drawn as **parallel** lines towards the surface, at equal intervals from each other.

Gravitational field strength

The gravitational field strength, *g*, is defined as the **gravitational force** experienced **per unit mass** by an object at that point in a gravitational field. It is a vector quantity, with the units Nkg^{-1} or ms^{-2}.

$$g\ \{gravitational\ field\ strength\} = \frac{F\ \{gravitational\ force\}}{m\ \{mass\ of\ the\ object\ in\ the\ field\}}$$

This equation is accurate as long as the mass of the object in the field is small enough that the **object's gravitational field** is **negligible** compared to the **external** gravitational field the object is in.

Newton's law of gravitation

The force between two point masses

Newton's law of gravitation states *two point masses attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of their separation.* This can be written to show that the gravitational force, *F*, between two objects is

$$F = \frac{-GMm}{r^2}$$

Where G is the gravitational constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$, M and m are the masses of the two objects, and r is the distance between their centres. The **negative sign** is used to show the **attractive nature** of the force.

Gravitational field strength for a point mass

As the gravitational field strength is given by $g = \frac{F}{m}$, the gravitational field strength for a point mass can be given by dividing the gravitational force between two point masses by the mass of the other point mass. This produces

$$g = \frac{GM}{r^2}$$

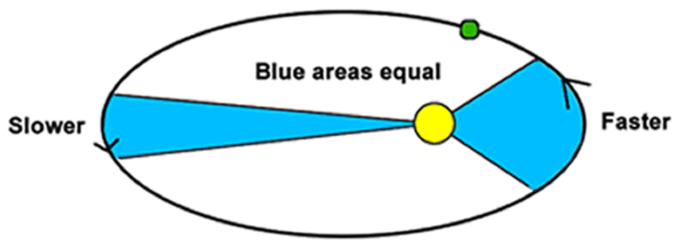
This shows that the field strength for an object, such as a planet, does not depend on the mass of the object in orbit around the planet, only on the **mass of the planet** and the **distance** between them.

When considering the gravitational field strength close to the earth's surface, the field can be modelled as uniform, and has the same value as the acceleration of free fall.

Planetary motion

Kepler's laws

Kepler's first law states *the orbit of a planet is an ellipse, with the sun at one focus.* The eccentricity of the ellipse is very low, so the motion can be **modelled** as **circular.**



Kepler's second law states *a line segment joining a planet and the sun sweeps out equal areas during intervals of equal time.* This is because the speed of the planet is not constant – the planet moves faster when it is closer to the sun.

Kepler's third law states *the square of the orbital period T is proportional to the cube of the average distance r from the sun.* This can be proved by considering the forces acting on the planet. Centripetal force is required to keep the planet in orbit, and this force is provided by the gravitational field of the sun. Because of this, we can **equate the formula** for centripetal force with the formula for gravitational force to get

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

We can rearrange this to produce

$$\frac{GM}{r} = v^2$$

As the velocity of an object in circular motion can be written as $v = \frac{2\pi r}{T}$, we can show that

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{t^2}$$

This can be rearranged to show that

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Where *T* is the period of orbit, *r* is the average distance between the planet and the sun (from centre to centre), *G* is the gravitational constant, and *M* is the solar mass. Because *G* and *M* are constants, this formula shows that T^2 is proportional to r^3.

Satellites

Satellites are objects that orbit other, larger objects. These can include natural satellites, like the moon, and also artificial satellites that humans have sent in to space. Satellites can be used for communications, scientific research, and Global Positioning Systems (GPS).

Geostationary satellites have an orbital period of one day. They travel in the **same direction** as the rotation of the Earth, **along the equatorial plane**. This means that they remain above the same point on the Earth's surface, making them useful for **communications and surveying**, as they provide continuous coverage.

Gravitational potential and energy

Gravitational potential

All masses attract each other, and it takes energy to move objects apart. **Gravitational potential**, V_g, is defined as the **work done per unit mass** to move an object to that point from infinity. The unit is Jkg^{-1}. At infinity, the field is negligible, so V_g is 0, which is its maximum value. At all other points, V_g is **negative**, which represents how **energy is required** to move the object out of the field.

V_g can be calculated using the formula

$$V_g = -\frac{GM}{r}$$

Where G is the gravitational constant, M is the mass the object is being moved away from (not the mass of the object itself) and r is the separation distance between the object and the mass.

Gravitational potential energy

Gravitational potential energy is the **work done** to move an object with mass *m* from infinity to a point in that field.

$$E = mVg, \text{since } Vg = -\frac{GM}{r}, E = -\frac{GMm}{r}$$

Escape velocity

For an object to **escape a gravitational field** produced by a mass *M*, the kinetic energy of the object at the start must be equal to or greater than the gravitational potential energy required to lift it to infinity. The escape velocity is the same for any object at that starting radius r, regardless of the mass *m* of the object.

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \text{ so } v = \sqrt{\frac{2GM}{r}}$$